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Solution-Responsive Particle Size Adaptivity in Lagrangian Vortex Particle Methods

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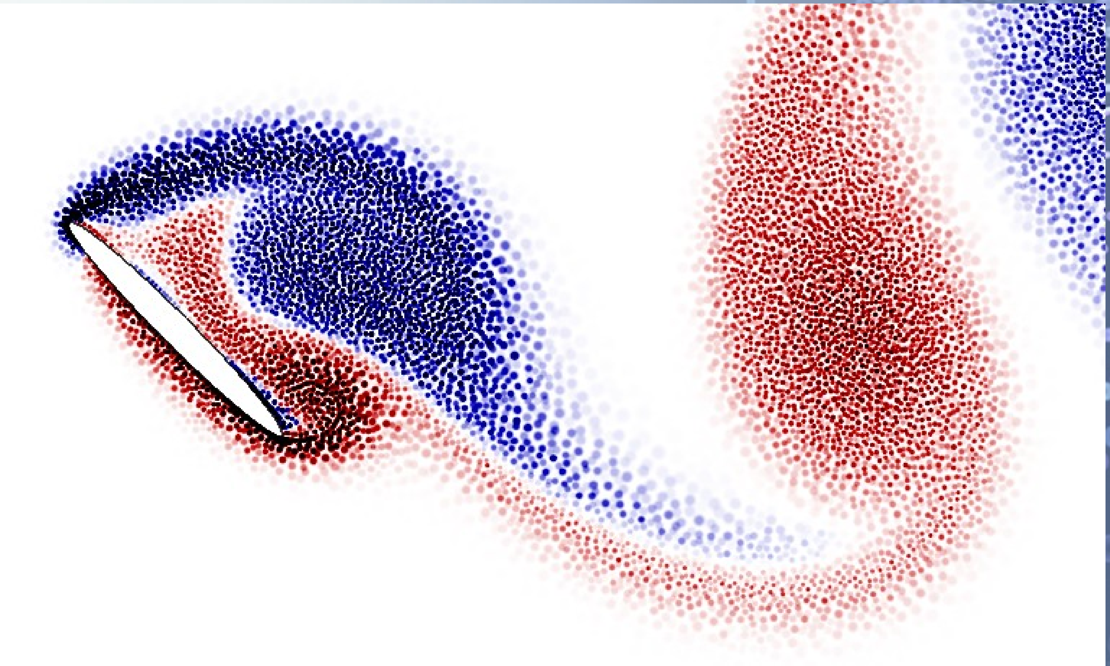
Outline

- Adaptivity and Diffusion in Particle Methods
- Overview of Vorticity Redistribution Method (VRM)
- Adaptive VRM
- Diffusion of a Point Vortex
- Re=9500 Cylinder
- Parameter Effects



Adaptivity in Particle Methods

- Naturally adaptive (particles only where vorticity)
- Cutoffs (distance or strength) [various]
- Merging/lumping [Spalart, Rossi]
- Regridding to AMR [Bergdorf *et al.*, Rasmussen *et al.*]
- Replacement onto alternative set [Reboux *et al.*]
- Higher-order shapes [Teng, Rossi, Häcki]

Diffusion in Particle Methods

- Random Walk [Chorin]
- Core-spreading [Kuwahara & Takami, Rossi]
- Particle Strength Exchange [Mas-Gallic]
- Vorticity Redistribution Method [Shankar & van Dommelen]
- Radial Basis Functions [Barba *et al*.]
- Finite-Difference after Regridding [Cottet]

Vorticity Redistribution Method

- Solve under-determined equations for fractions of circulation to contribute to neighbors (f_i)
- Add new neighbors if no solution is available

$$\tilde{x}_j = \frac{x_j - x_i}{h_\nu} \qquad \sum_j f_j = 1 \qquad \sum_j \tilde{x}_j^2 f_j = 2$$

$$\tilde{y}_j = \frac{y_j - y_i}{h_\nu} \qquad \sum_j \tilde{x}_j f_j = 0 \qquad \sum_j \tilde{x}_j \tilde{y}_j f_j = 0$$

$$h_\nu = \sqrt{\frac{\Delta t}{\mathrm{Re}}} \qquad \sum_j \tilde{y}_j f_j = 0 \qquad \sum_j \tilde{y}_j^2 f_j = 2$$

Adaptive VRM

- Second moment conservation must account for core radius (σ) at start and end of diffusion step

$$\sum_j \left(\tilde{x}_j^2 + C_2 \left(\frac{\tilde{\sigma}_j}{h_\nu} \right)^2 \right) f_j = 2 + C_2 \left(\frac{\sigma_i}{h_\nu} \right)^2$$

$$\sum_j \tilde{x}_j \tilde{y}_j f_j = 0$$

$$\sum_j \left(\tilde{y}_j^2 + C_2 \left(\frac{\tilde{\sigma}_j}{h_\nu} \right)^2 \right) f_j = 2 + C_2 \left(\frac{\sigma_i}{h_\nu} \right)^2$$

Adaptive VRM, Continued

- How to set new particle sizes/radii (σ)?
- Adapt and diffuse thresholds, and their metric
  - Metric is particle circulation*
  - If metric is above adapt threshold, allow constant-radius VRM
  - If metric is below adapt threshold, allow growth+VRM
  - If metric is below diffuse threshold, allow only growth
- Radius lapse rate (C_{lapse})
  - Limits growth to a given spatial gradient of core radius

Adaptive VRM, Continued

- Calculation of new particle core sizes (σ_i)

$$\sigma_{i,lapse} = \min\left[\sigma_j + C_{lapse} \,||\vec{x}_i - \vec{x}_j||_2\right]_{j \neq i}$$

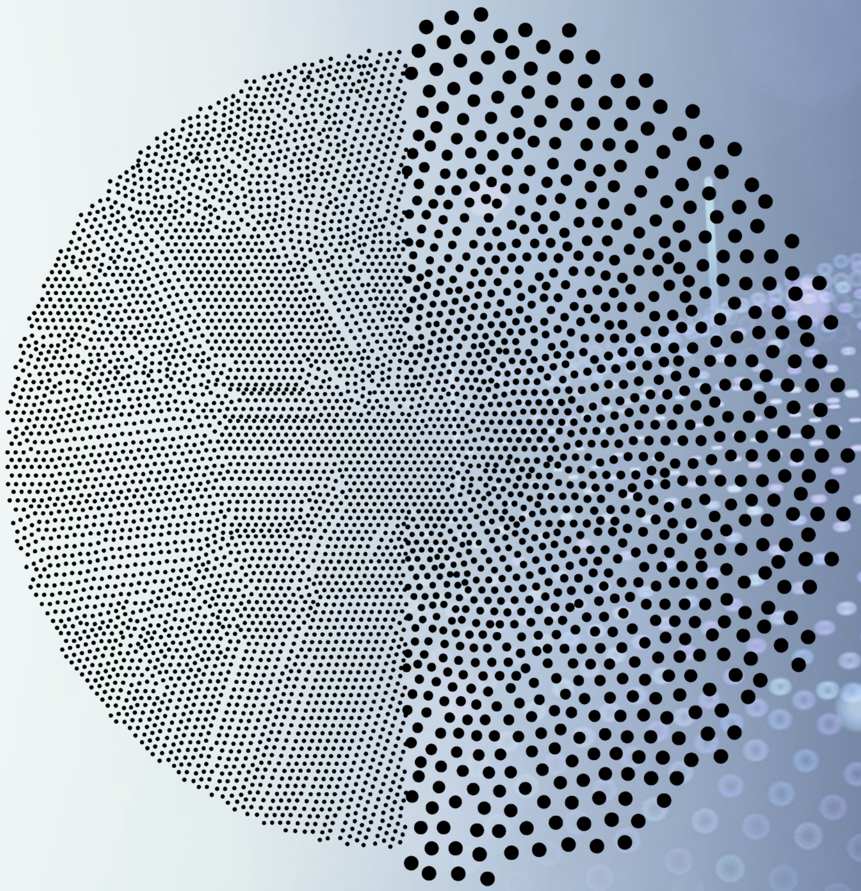
$$\sigma_{i,grow} = \sqrt{\sigma_i^2 + 2\, h_\nu^2 / C_2}$$

$$\sigma_{i,test} = \min\left[\sigma_{i,lapse}, \sigma_{i,grow}\right]$$

$$\tilde{\sigma}_i = \begin{cases} \frac{3\sigma_i + \sigma_{i,test}}{4}, & \text{if } \sigma_{i,test} > \sigma_i \\ \max[\sigma_{i,test}, 0.9\,\sigma_i], & \text{if } \sigma_{i,test} < \sigma_i \end{cases}$$

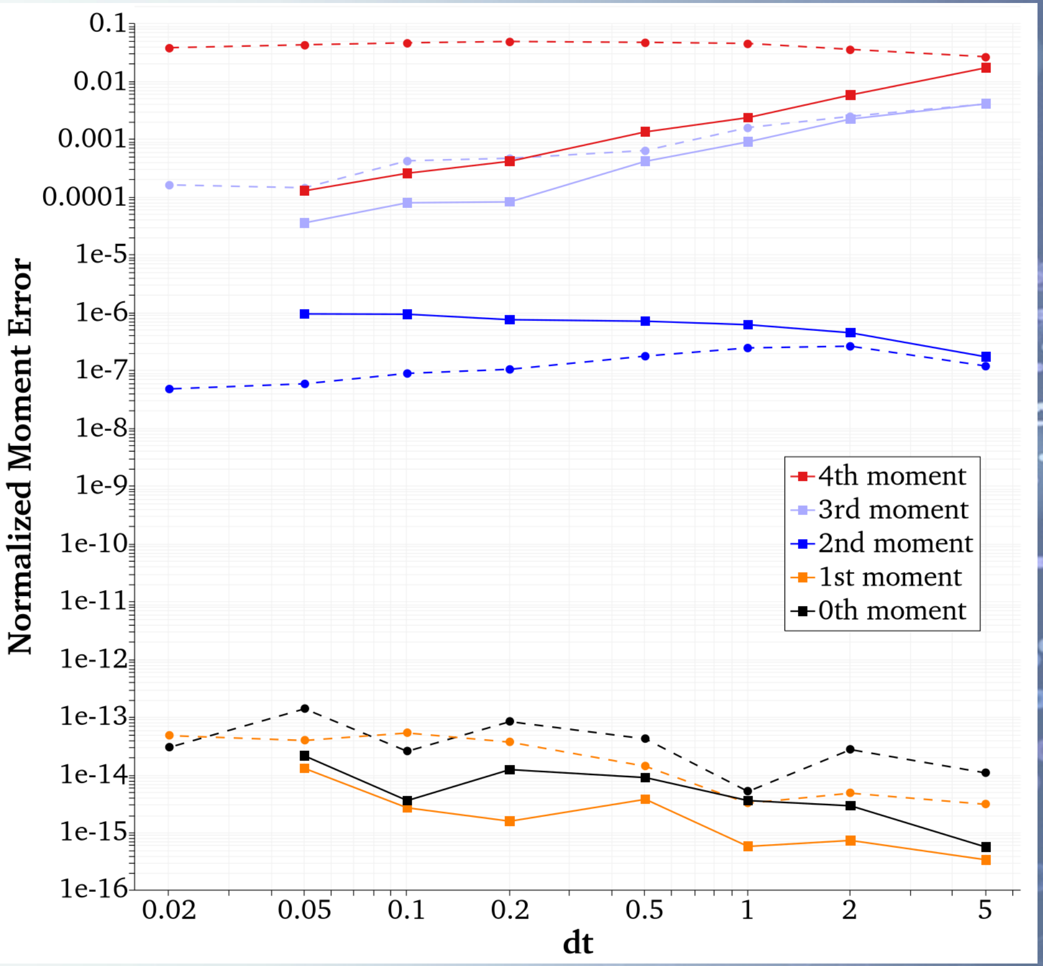
Point Diffusion

- Diffusion only, no convection
- 250 Steps of VRM
- No adaptivity: N_v=8,105
- With adapt threshold 10^{-3}, diffuse threshold 10^{-6}, and lapse rate 0.1, N_v=2,458



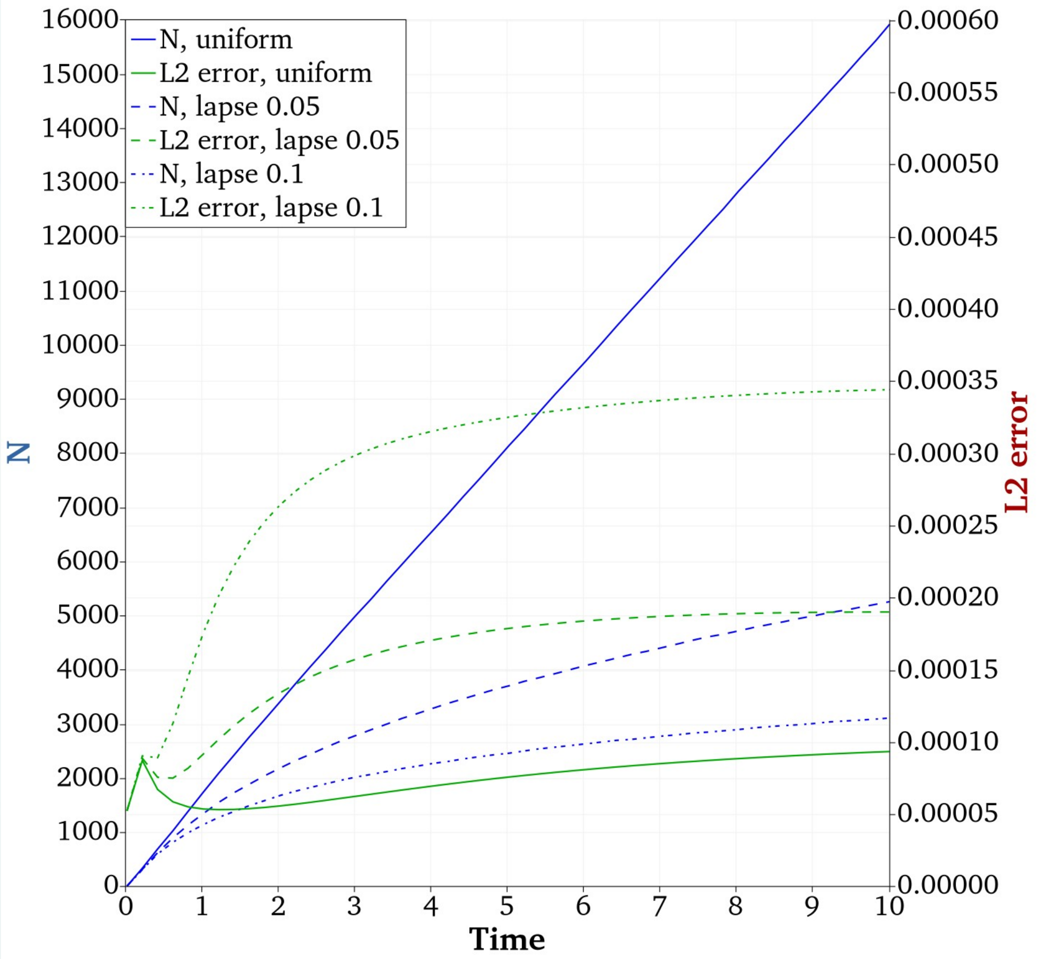
Point Diffusion, Continued

- Unit-strength point vortex, unit diffusivity
- Conserve moments [0..2]
- Run to t=100
- Compare errors vs. theoretical moments

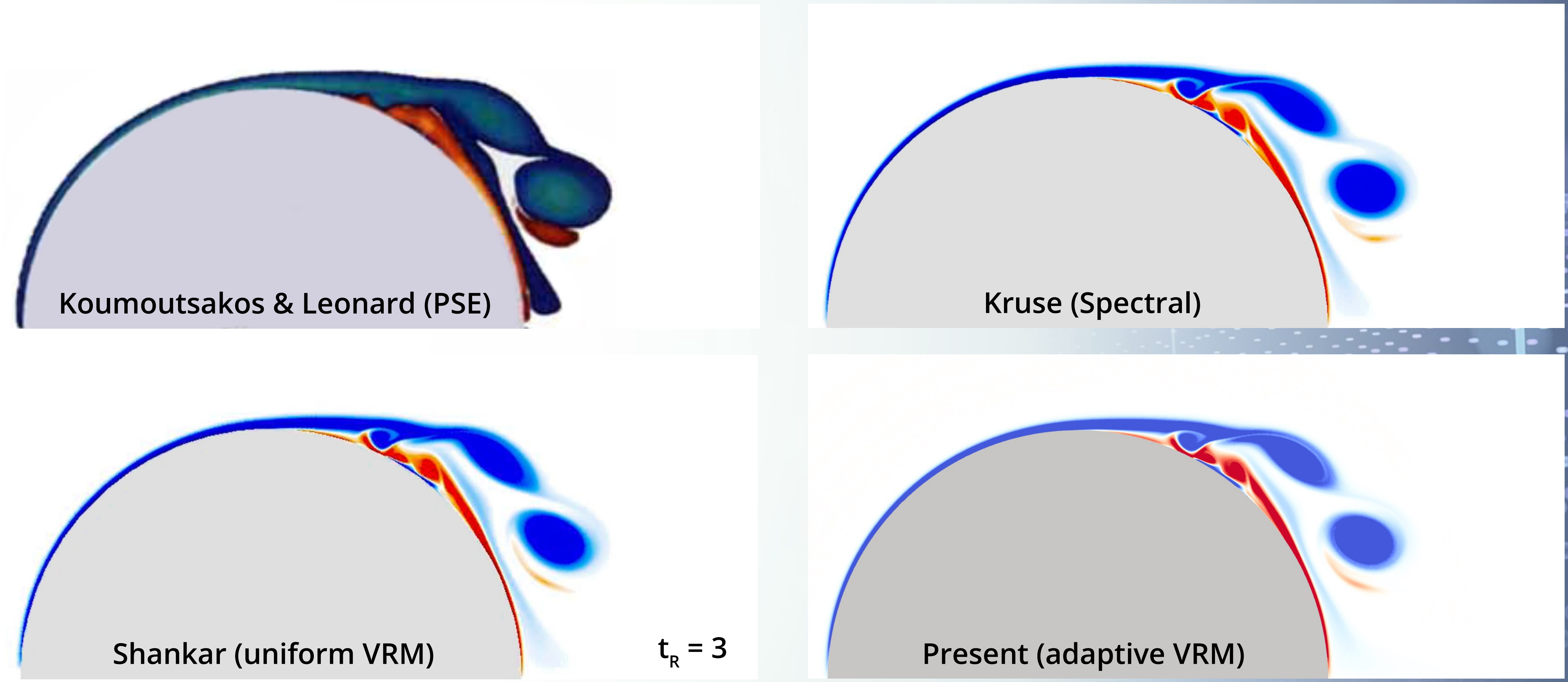


Point Diffusion, Continued

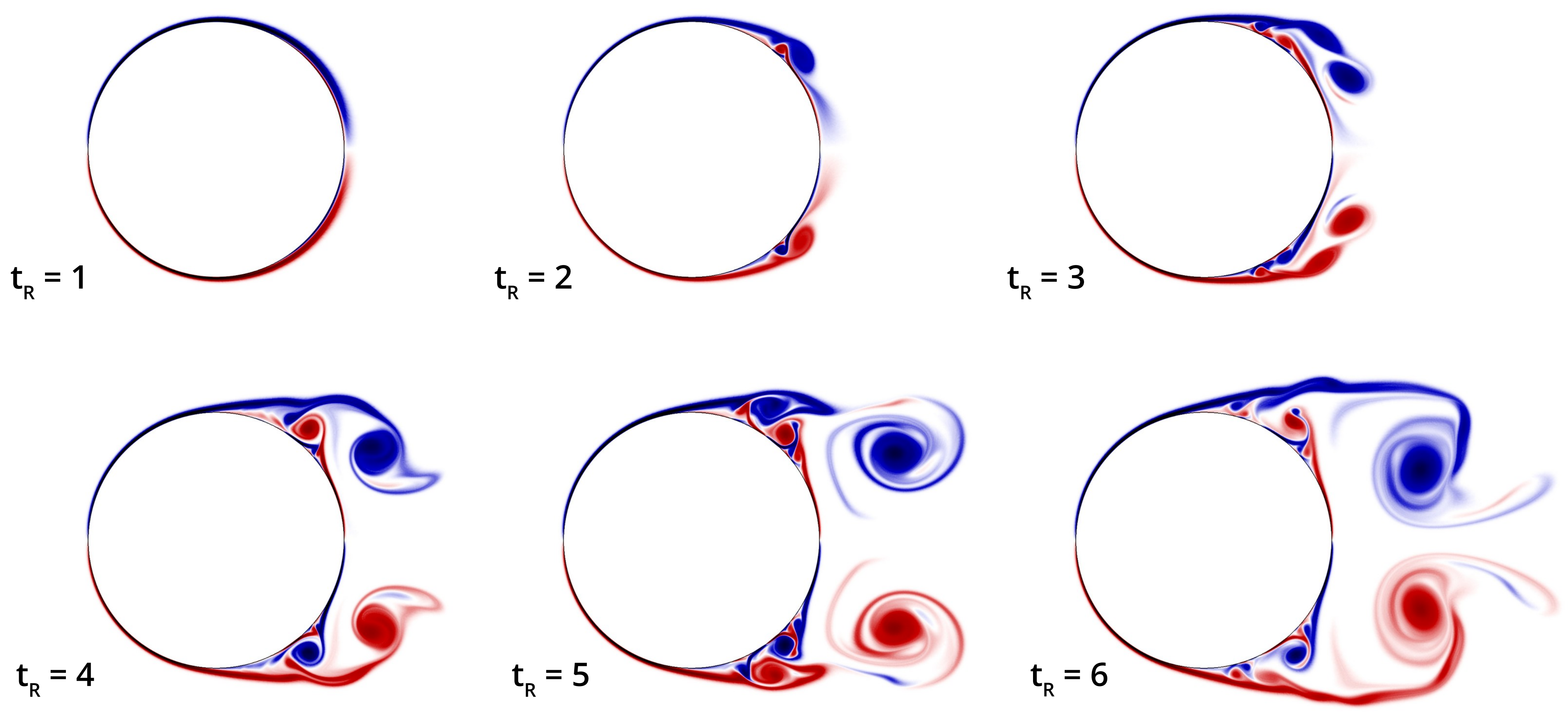
- Re=1000, Δt=0.02
- Pointwise L_2 error vs. time
- Demonstrates trade-off between adaptivity and accuracy



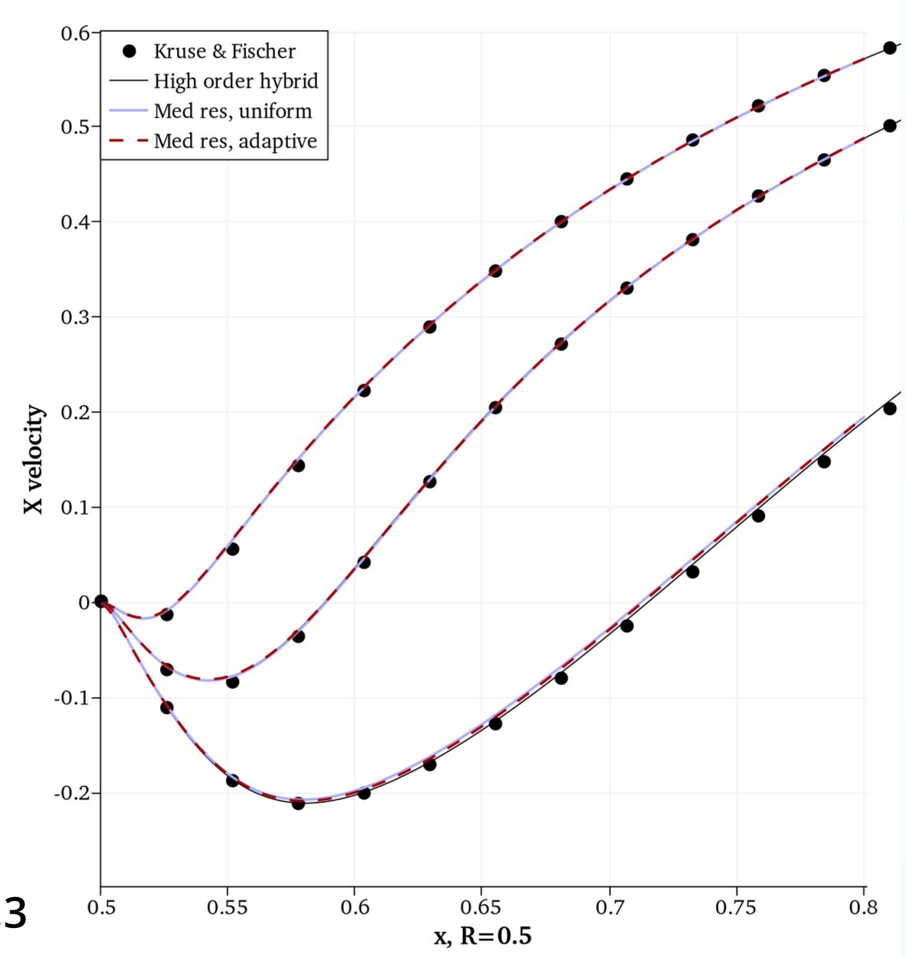
Re=9500 Cylinder



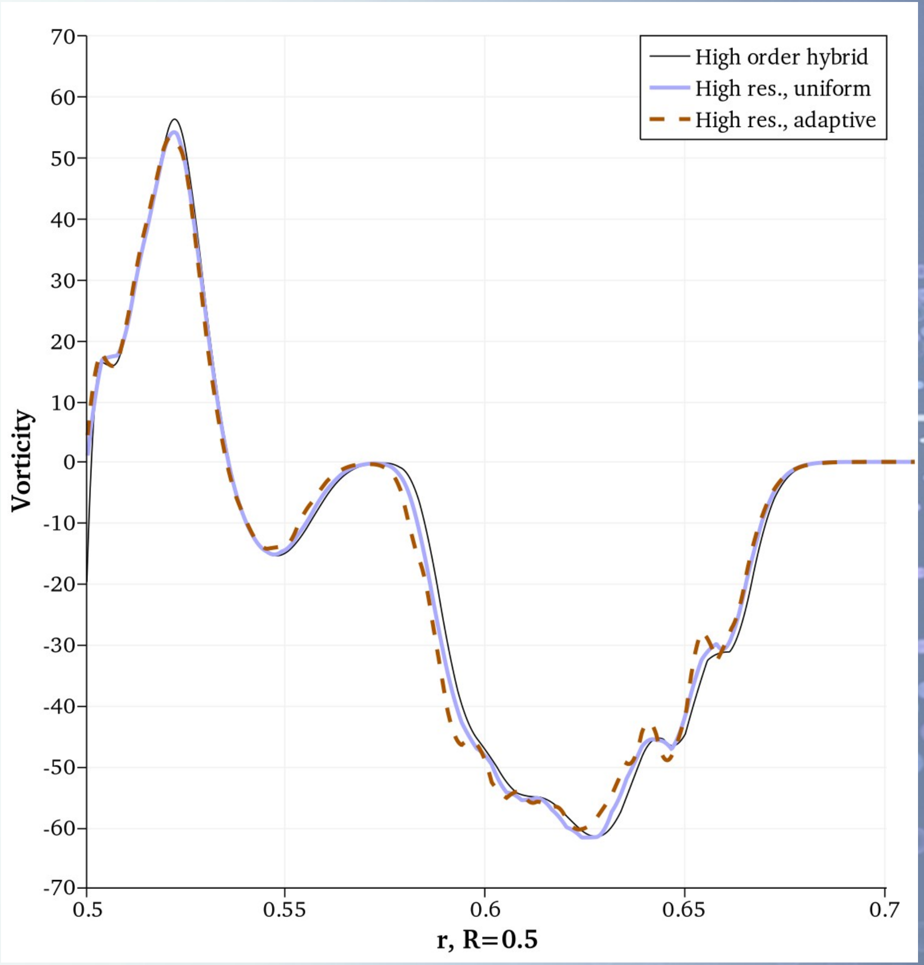
Re=9500 Cylinder Results



Re=9500 Cylinder Results

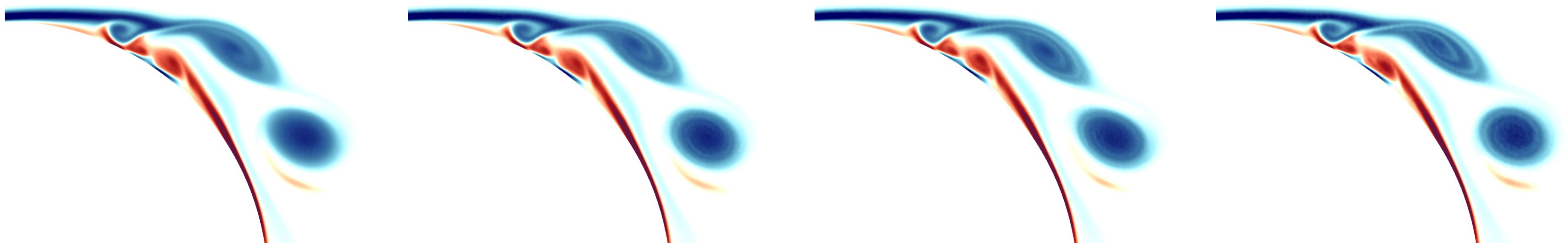


t_R = 1,2,3



t_R = 3

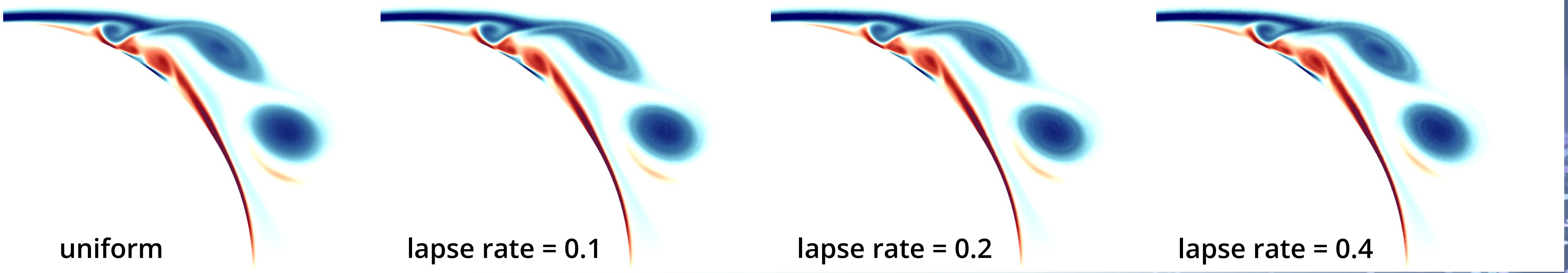
Effect of Adaptivity Threshold



No adapt, 10^{-5} diffuse | 10^{-3} adapt, 10^{-6} diffuse | 10^{-2} adapt, 10^{-4} diffuse | 10^{-1} adapt, 10^{-2} diffuse

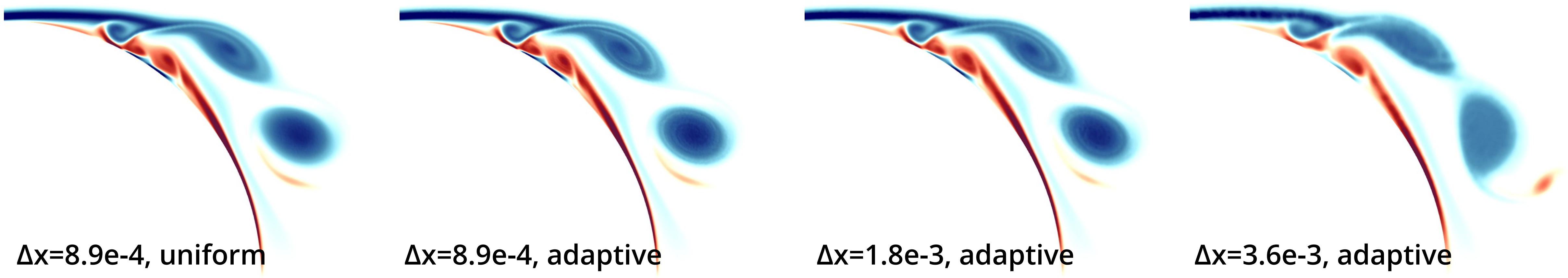
- Vorticity at $t_R=3$, $\Delta t_R=0.01$
- Radius lapse rate = 0.15
- N_v = 155k, 91k, 81k, 55k

Effect of Radius Lapse Rate



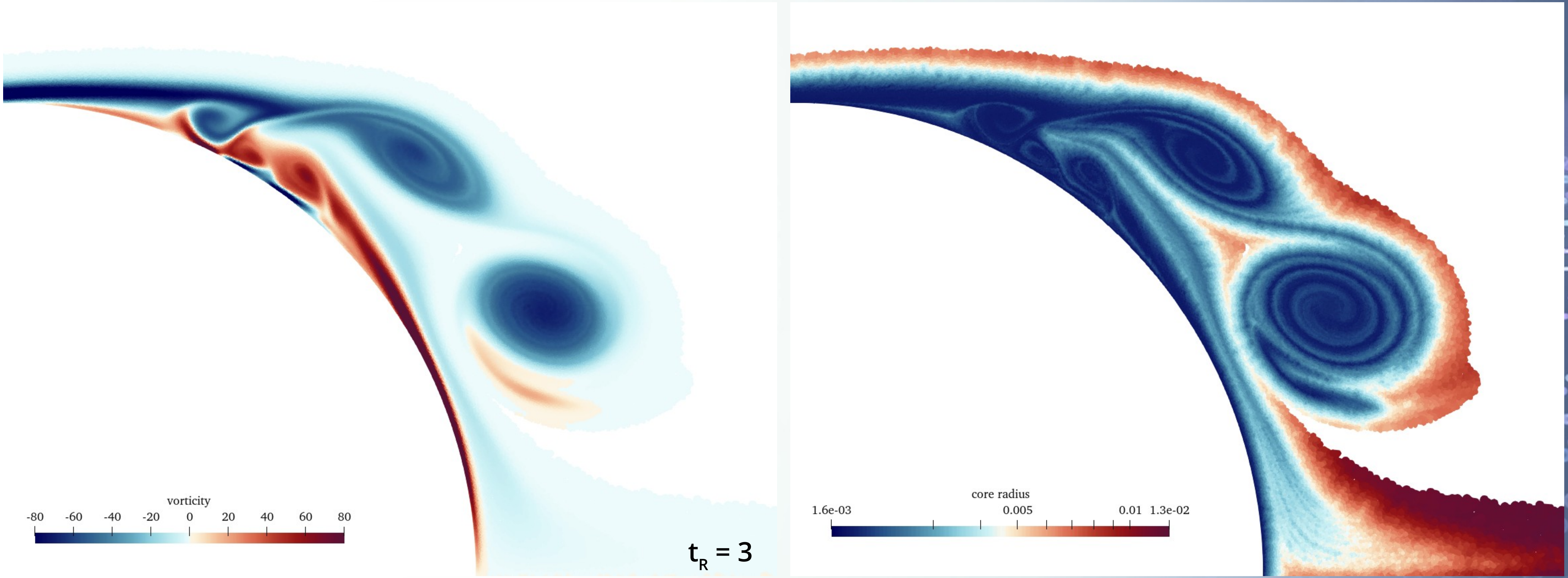
- Vorticity at t_R=3, Δt_R=0.01
- 10^{-2} adapt, 10^{-4} diffuse
- N_v = 155k, 87k, 77k, 75k

Effect of Resolution

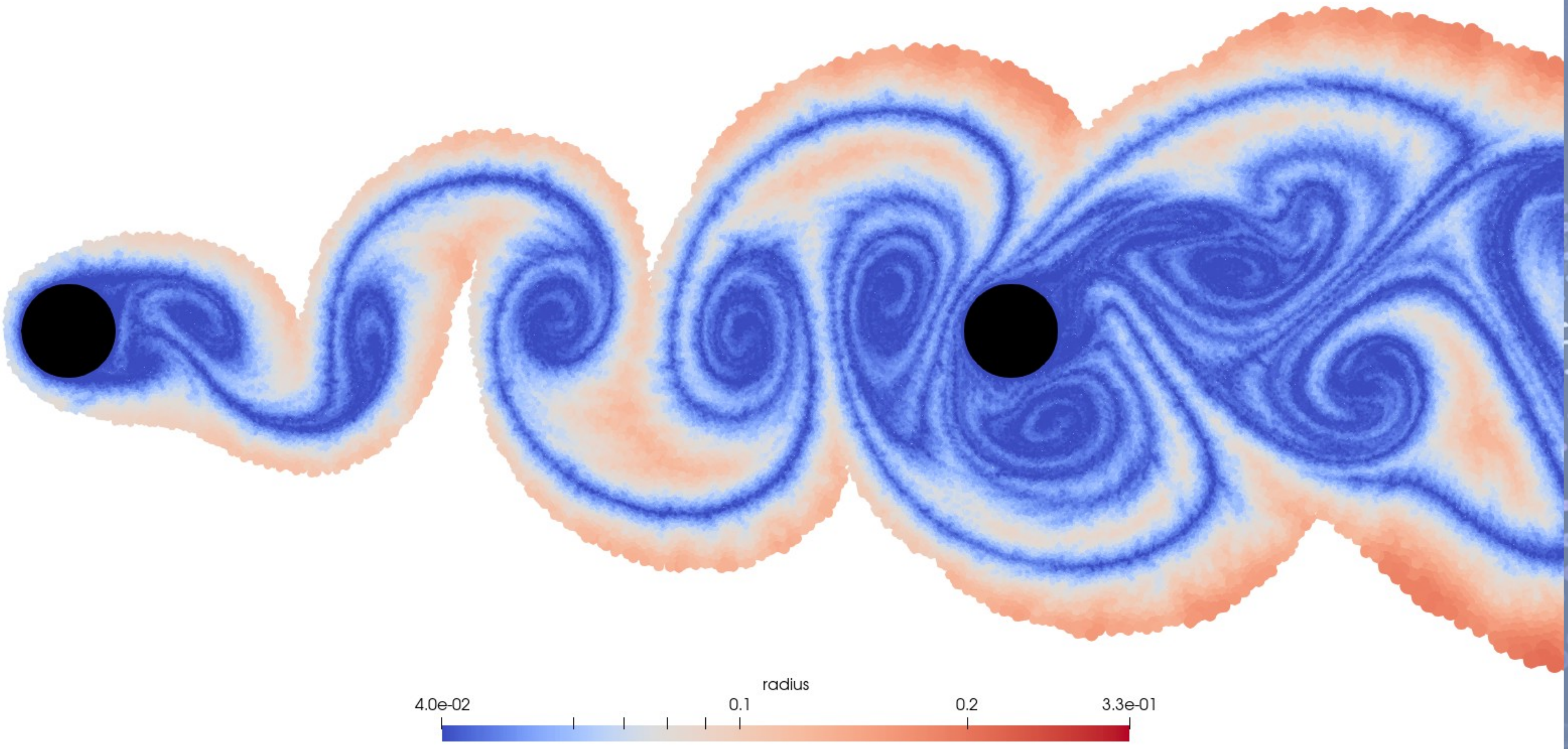


- Vorticity at t_R=3, Δt_R = 0.0025, 0.0025, 0.01, 0.04
- 10^{-2} adapt, 10^{-4} diffuse, lapse rate = 0.15
- N_v = 598k, 240k, 81k, 27k

Re=9500 Cylinder



Re=300 Tandem Cylinders



Conclusions

- True solution adaptivity with no *a priori* guidance
- Particles can take any radius (not quantized)
- 2-D version offers 50-75% reduction of N_v
- Small overhead is far outweighed by lower N_v
- Still works with global spatial adaptivity schemes

Next Steps

- 3-D tests show even larger reduction of N_v
- Investigate new adaptivity metrics (shear rate)
- Apply to Hybrid HO Eulerian-Lagrangian Method

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