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# A Hybrid High-Order Vorticity-Based Eulerian and Lagrangian Vortex Particle Method, the 2-D Case

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# Outline



- Lagrangian Vortex Particle Method
- Vorticity-based High Order Eulerian method
- Hybridization
- Re=1000 Lid-driven cavity
- Re=9500 Cylinder
- Parameter effects

# Lagrangian Vortex Particle Methods



- Discretize vorticity onto radially-symmetric particles; scalar circulation in 2-D, vector strength in 3-D
- Solve vorticity equation in Lagrangian sense
- Biot-Savart or Particle-Grid methods for Velocity
- Boundary Element Methods solve for unknown singularity distributions on solid boundaries

# High-Order Eulerian Method



- Compact, high-order, discontinuous spectral difference method based on Huynh's flux reconstruction [2007,9]
- Vorticity-streamfunction variables
- Quad elements with geometry order 1→3, solution discretization order 1→4
- See companion paper FEDSM2021-63916

# High-Order Eulerian, Continued



- Eulerian time step can use explicit R-K orders1→4
- Each R-K substep performs the following:
  1) Solve *Poisson problem* for streamfunction, then u=grad $\psi$
  2) Compute *velocity jump* at solid boundaries
  3) Compute *diffused flux*, walls use Neumann, open Dirichlet
  4) Compute *convected flux*, all boundaries use Dirichlet

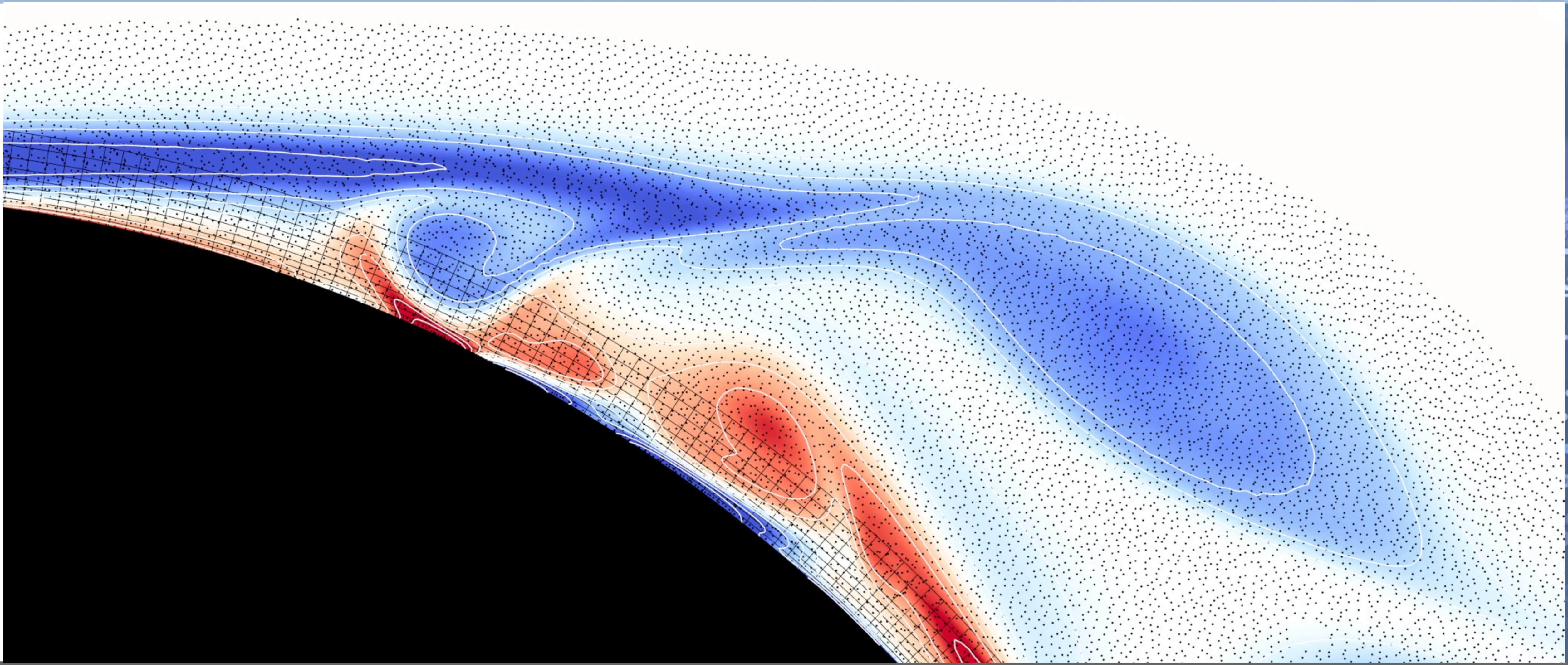
# Hybridization



- *Eulerian* solver provides solution near *boundaries*
- *Lagrangian* solver provides solution *elsewhere*
- *Lagrangian* solution provides spatial and temporal *boundary conditions* for Eulerian solver
- Eulerian solution feeds back to Lagrangian solution in an overlap zone near the body
- No iteration required

# Hybridization - Visually





# Hybridization - Overlap

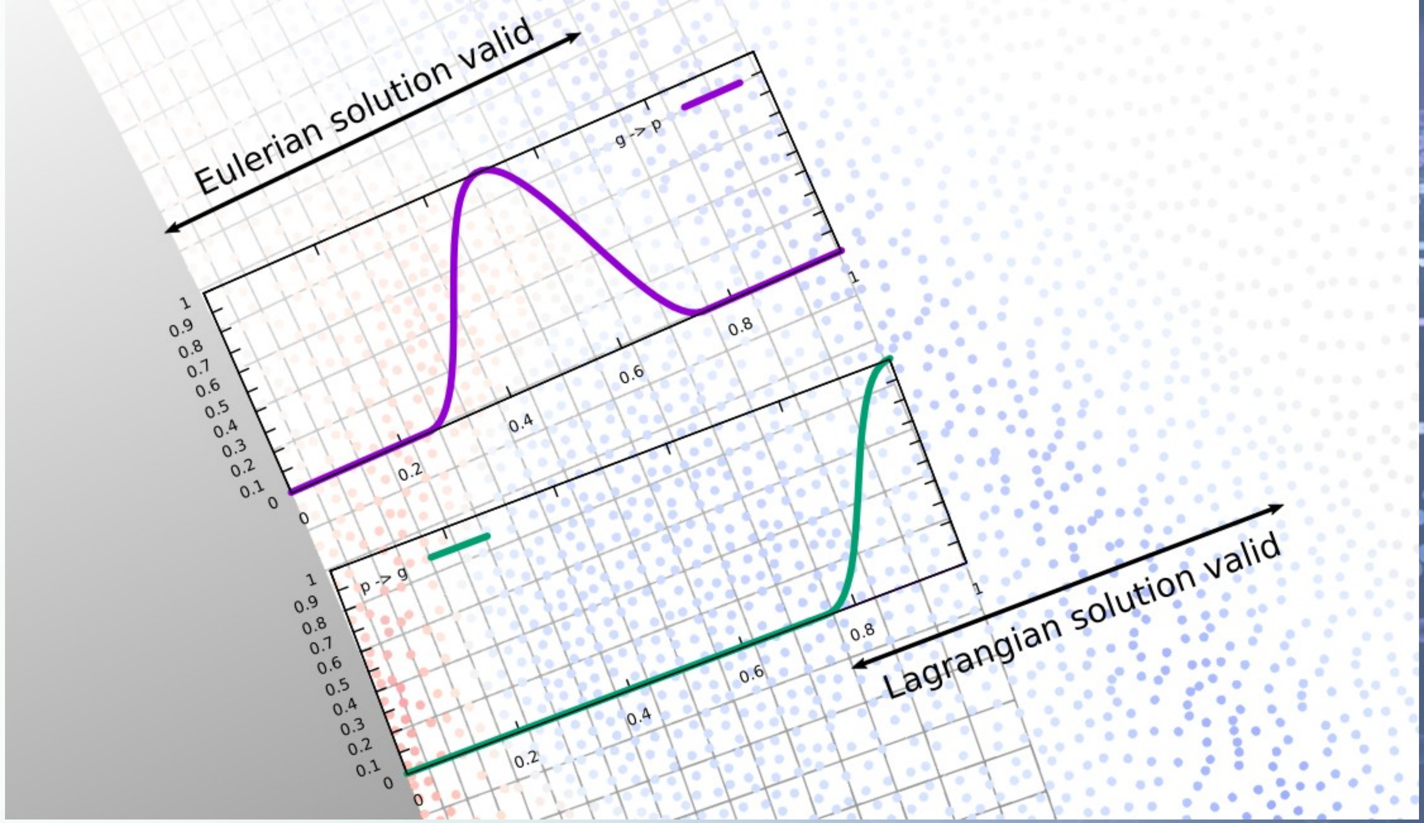


- Particle→grid and grid→particle operators

$$\hat{d} = \frac{d_{wall}}{d_{wall} + d_{open}}$$

$$\phi_{g\to p} = \begin{cases} \cos^2(2\pi\hat{d}) & \text{if } 0.25 < \hat{d} < 0.75 \\ 0 & \text{otherwise} \end{cases}$$

$$\phi_{p\to g} = \begin{cases} \cos^2(2\pi\hat{d}) & \text{if } \hat{d} > 0.75 \\ 0 & \text{otherwise} \end{cases}$$



# Hybridization – Time Step



- One outer (Lagrangian) time step performs:
  1) Half-step of diffusion (Vorticity Redistribution Method)
  2) 2nd Order Runge-Kutta convection step
  3) Second half-step diffusion
  4) Perform *hybrid update*

# Hybridization – Hybrid Update



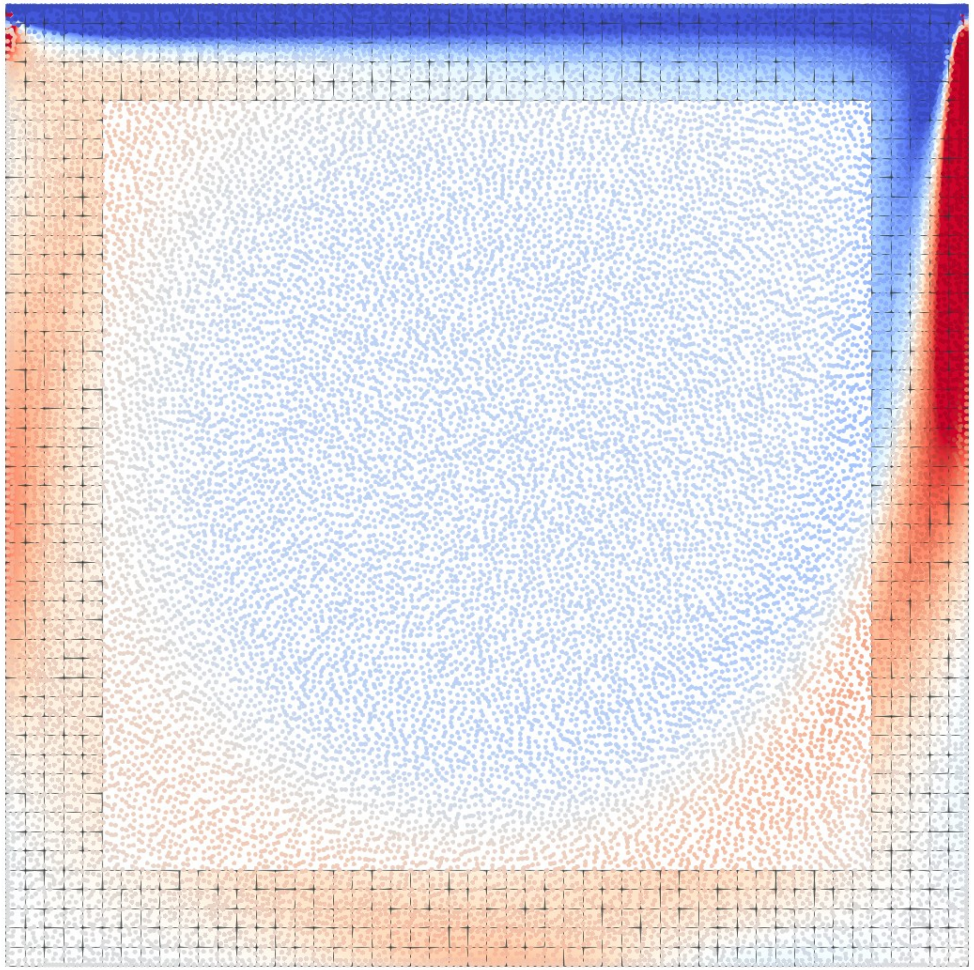
1. Lagrangian sends data to Eulerian:
   - ω and u at open boundary nodes at t and t+Δt
   - ω and p→g weights at all internal nodes at t and t+Δt
2. Eulerian system advances to time t+Δt
   - Set open BCs using linear interpolation from above
   - Advance Eulerian vorticity transport equations
   - Adjust ω toward Lagrangian vorticity using p→g weights

# Hybridization – Hybrid Update

3. Eulerian sends new ω back to Lagrangian
4. Lagrangian system updates ω at time t+Δt
   - ω at all Eulerian solution nodes compared to ω evaluated from Lagrangian solution alone to find ω deficit
   - The ω deficit is scaled by g→p weights and element areas
   - These are turned into new Lagrangian vortex particles
   - This process repeats 10 times or to $10^{-4}$ change

# Re=1000 Lid-Driven Cavity

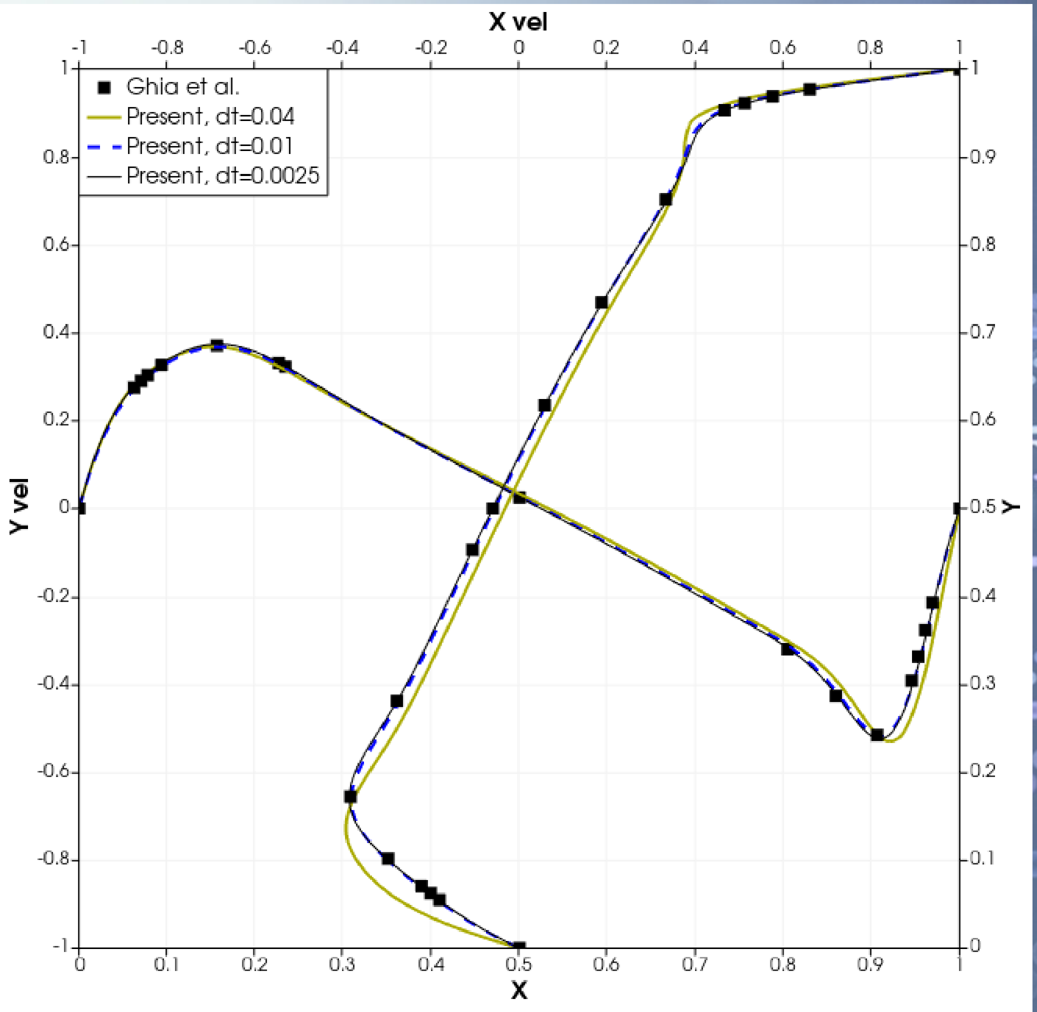
- Canonical 2-D test
- Compare results to steady-state solutions from Ghia *et al*. [1982] and Erturk *et al*. [2005]
- Medium resolution case: $\Delta x=0.02$, grid thickness=0.1



# Lid-Driven Cavity, Continued



- Results at t=75
- Visually, Ghia, Erturk, and our medium-res results look identical



# Lid-Driven Cavity, Continued

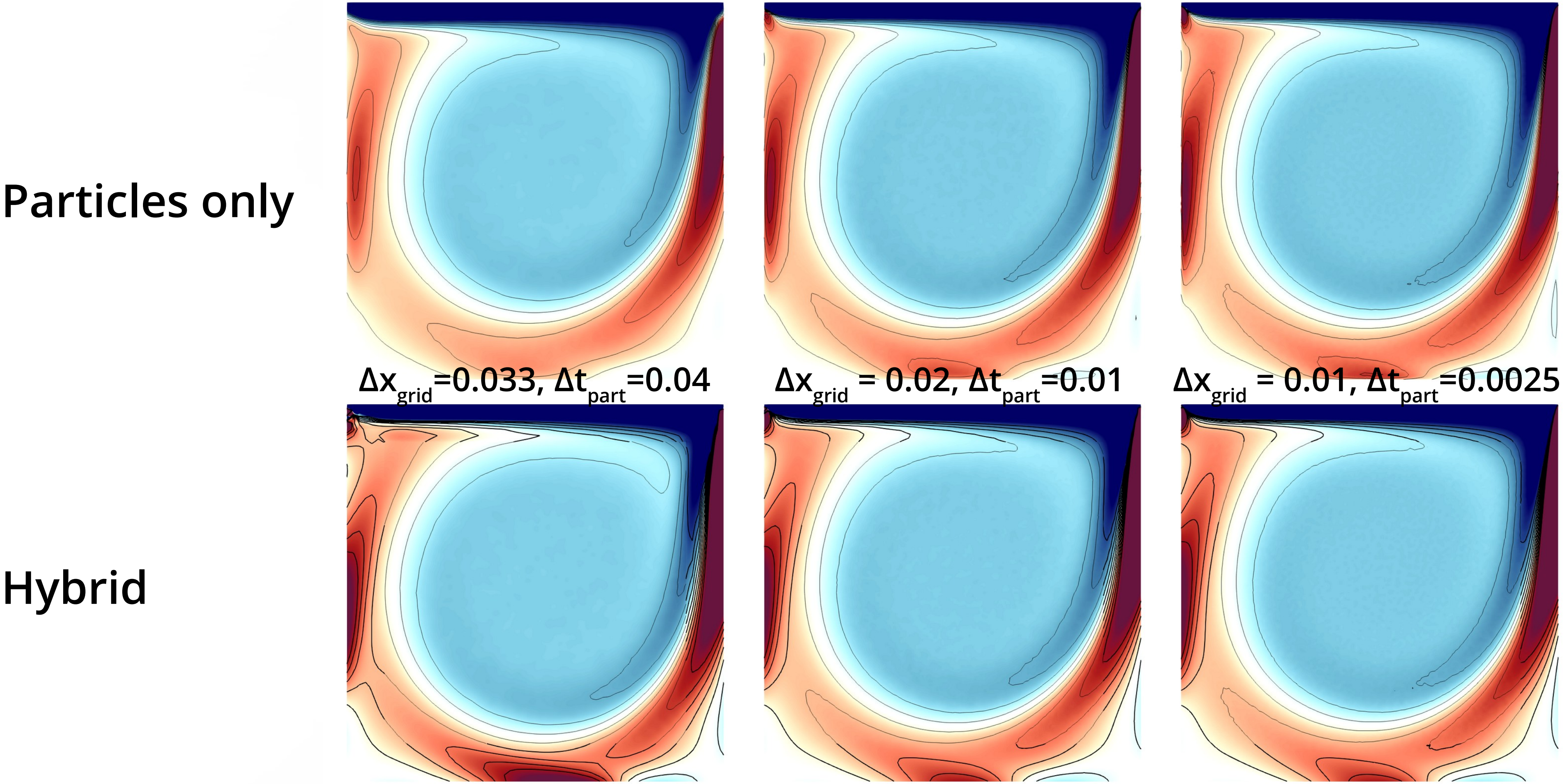


- Numerically closer to Erturk *et al*.
- 10 Eulerian substeps per Lagrangian step
- Dramatically fewer elements

| Parameter | Erturk *et al.* | Present Method | | |
|---|---|---|---|---|
| $\Delta t_{grid}$ | $\infty$ | 0.004 | 0.001 | 0.00025 |
| $\Delta x_{grid}$ | $0.001\bar{6}$ | $0.0\bar{3}$ | 0.02 | 0.01 |
| $N_{grid}$ | 361,201 | 324 | 900 | 3,600 |
| $\Delta t_{part}$ | - | 0.04 | 0.01 | 0.0025 |
| $\Delta x_{part}$ | - | 0.01549 | 0.00775 | 0.00387 |
| $u_{x,y=0.18}$ | -0.3869 | -0.3659 | -0.3792 | -0.3847 |
| $u_{x,y=0.94}$ | 0.4276 | 0.3901 | 0.4177 | 0.4251 |
| $u_{y,x=0.15}$ | 0.3756 | 0.3676 | 0.3665 | 0.3732 |
| $u_{y,x=0.91}$ | -0.5263 | -0.5216 | -0.5169 | -0.5241 |

# Lid-Driven Cavity, Continued





# Re=9500 Cylinder

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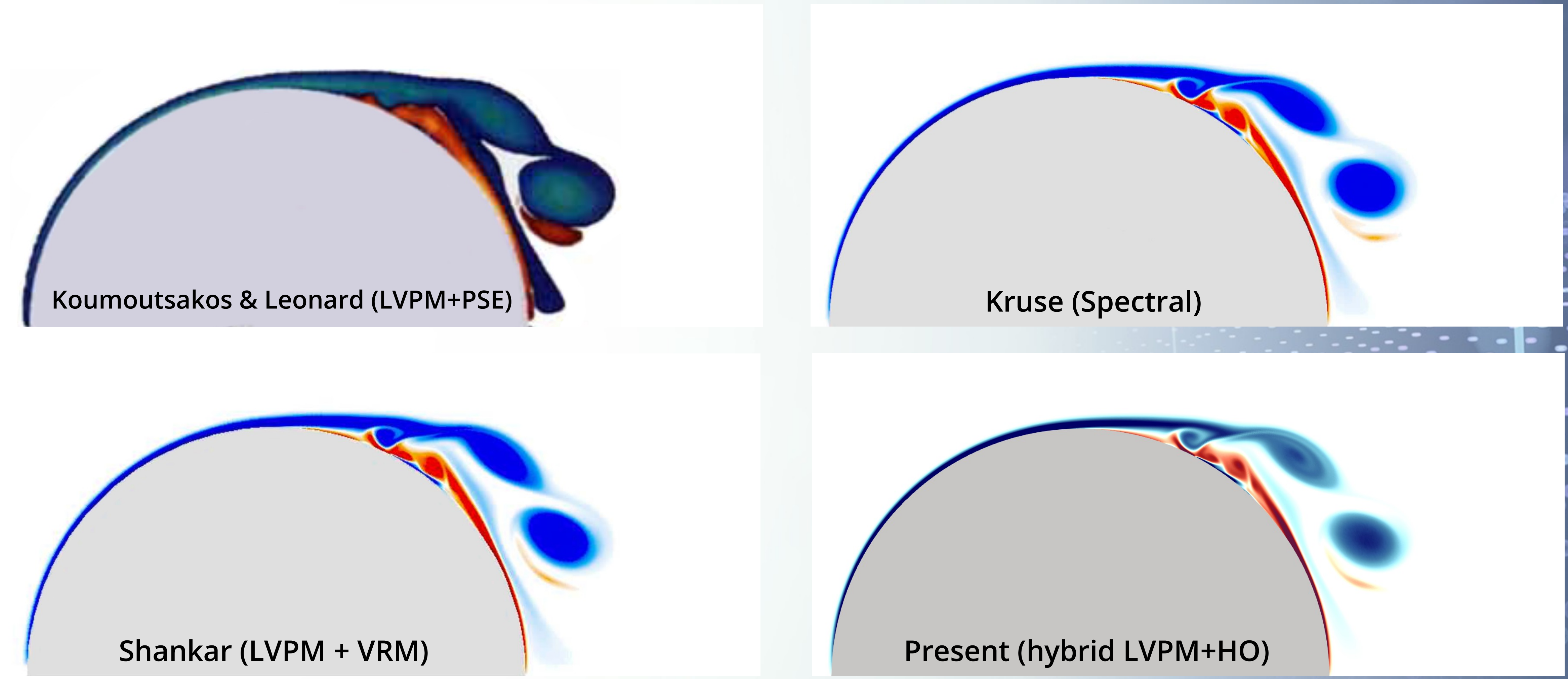
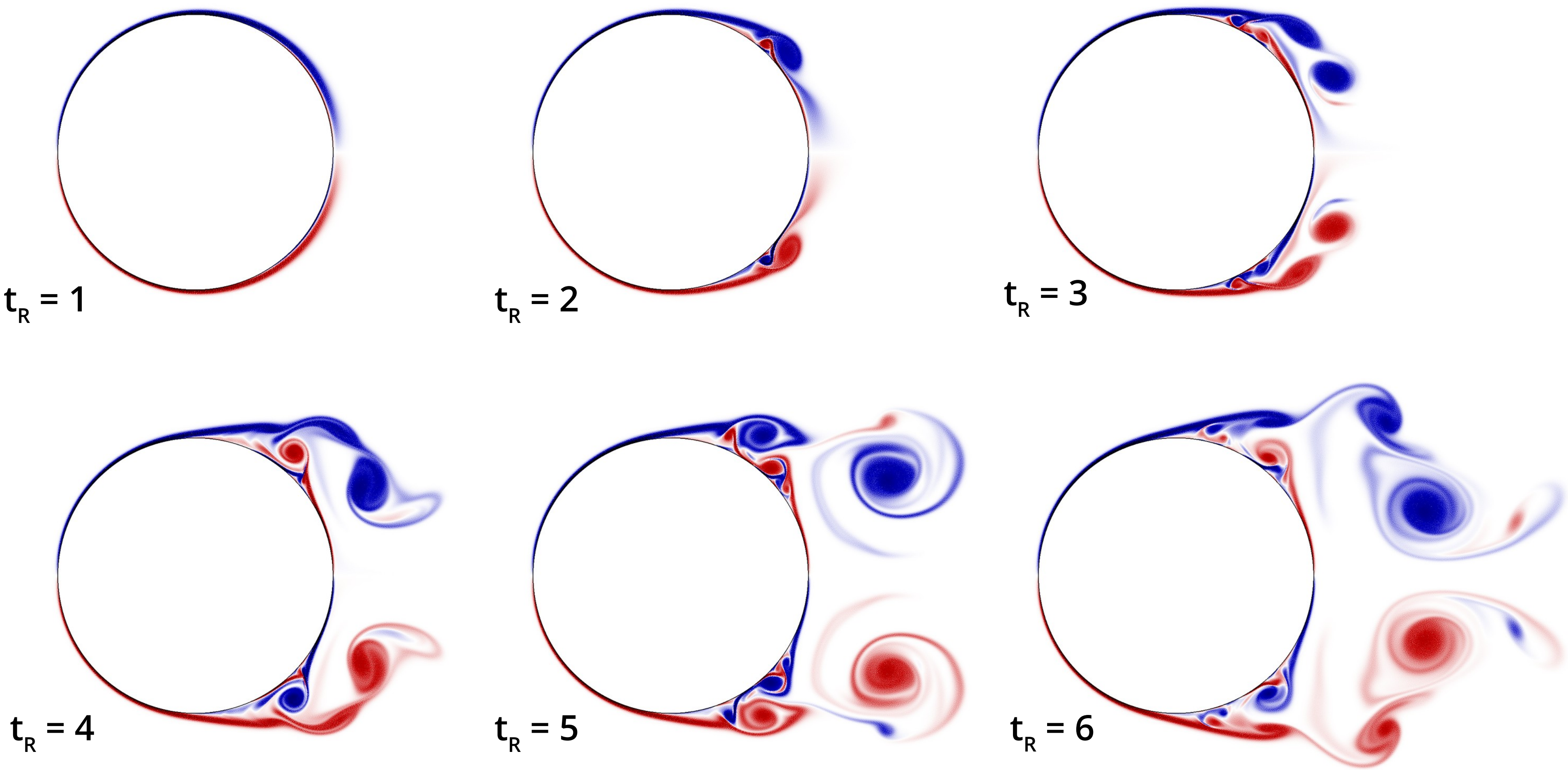


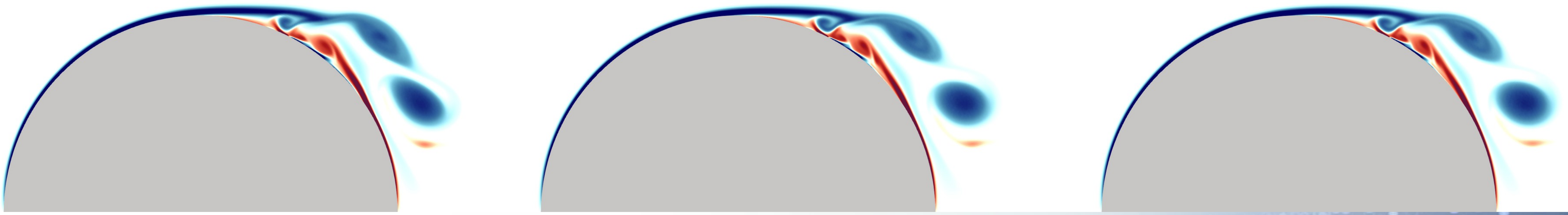
# Re=9500 Cylinder Results





# Effect of Cell Order





Hybrid, 12x864 2nd order

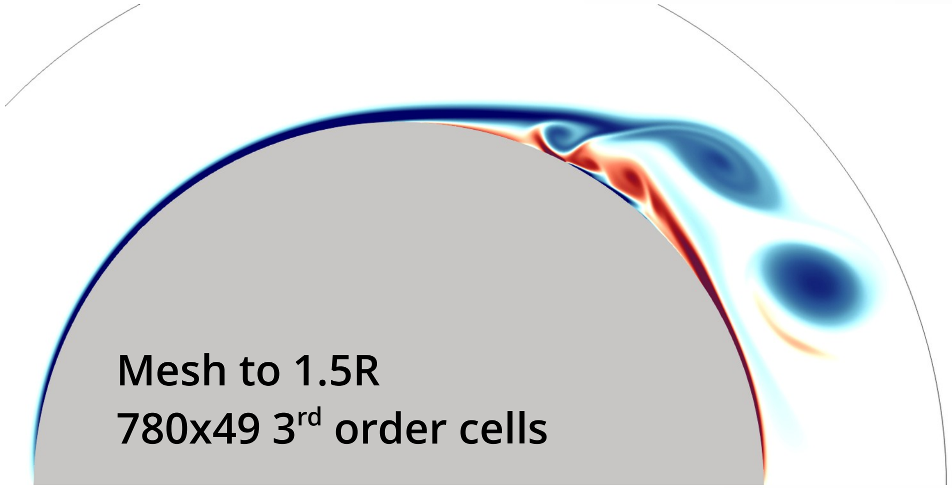
Hybrid, 8x576 3rd order

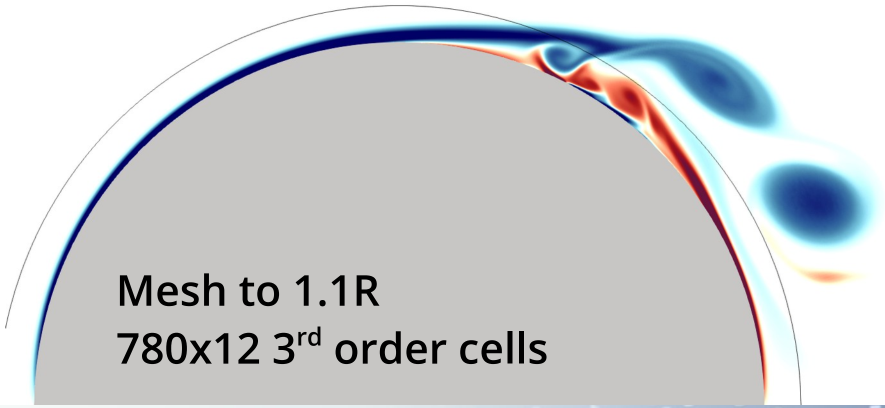
Hybrid, 6x432 4th order

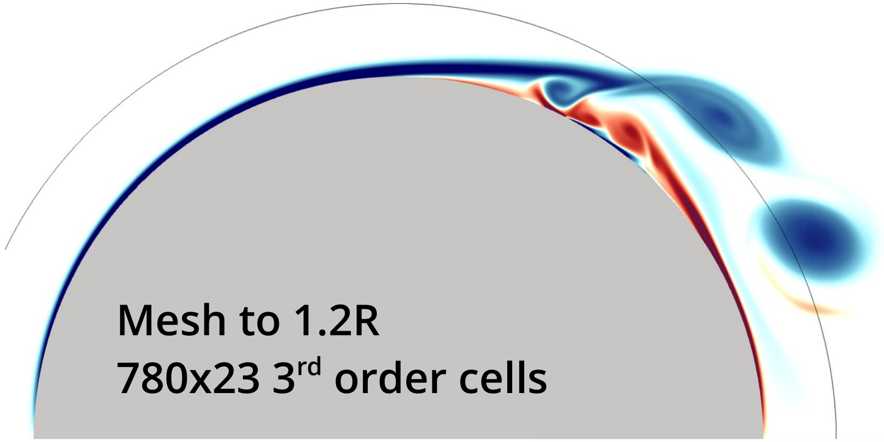
- Vorticity at $t_R$=3, $\Delta t_{R,particle}$=0.01, $\Delta t_{R,grid}$=0.001
- Eulerian mesh extends to 1.1R
- $N_v \approx$ 150k at $t_R$=3

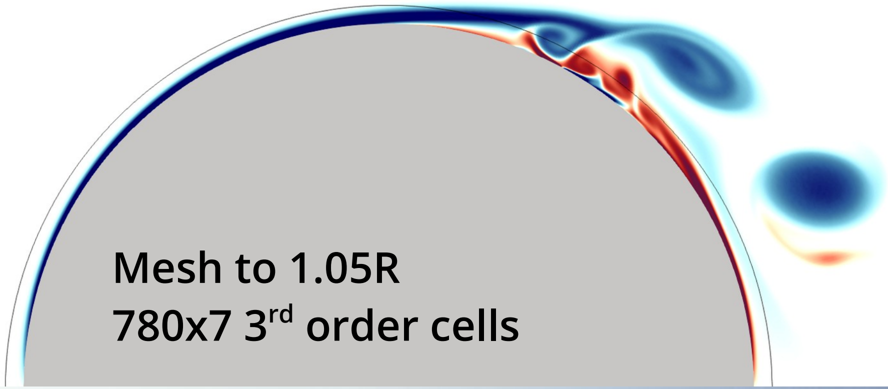
# Effect of Size of Eulerian Region





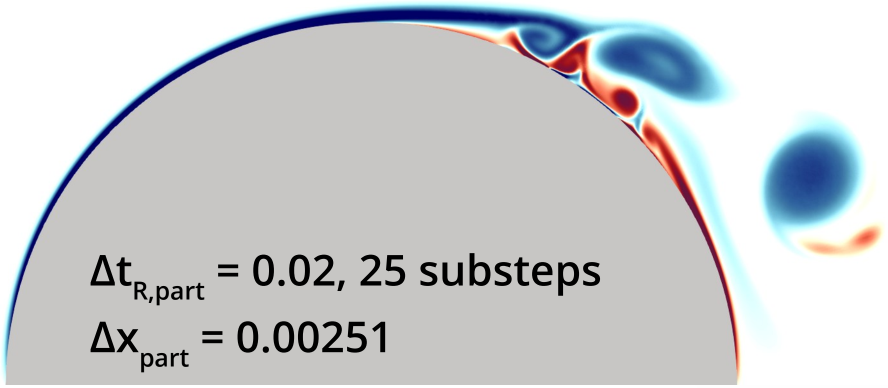



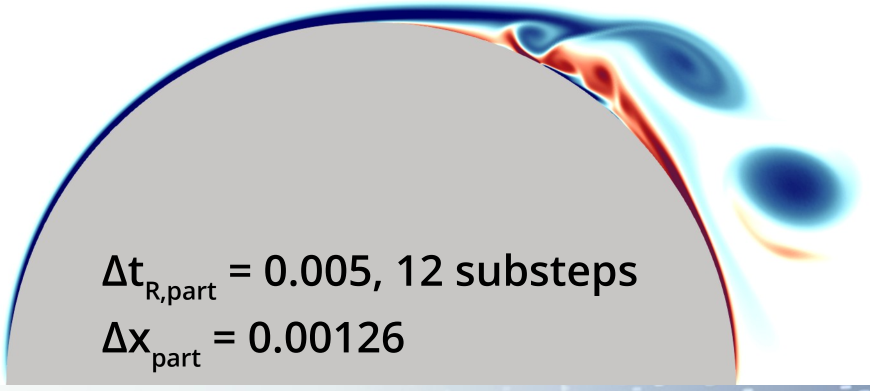



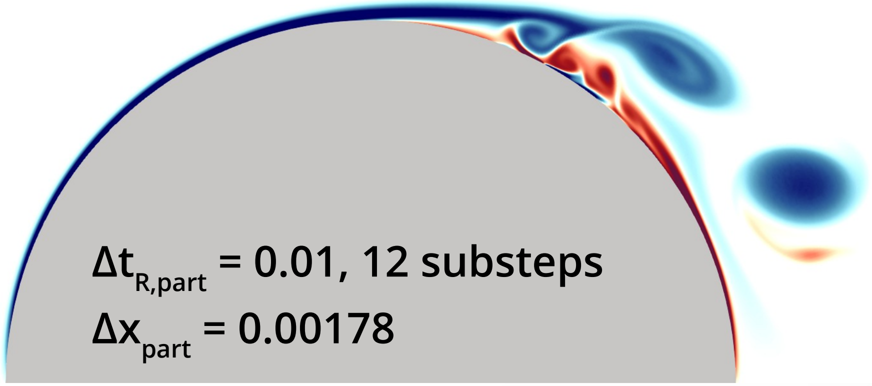
# Effect of Particle Resolution

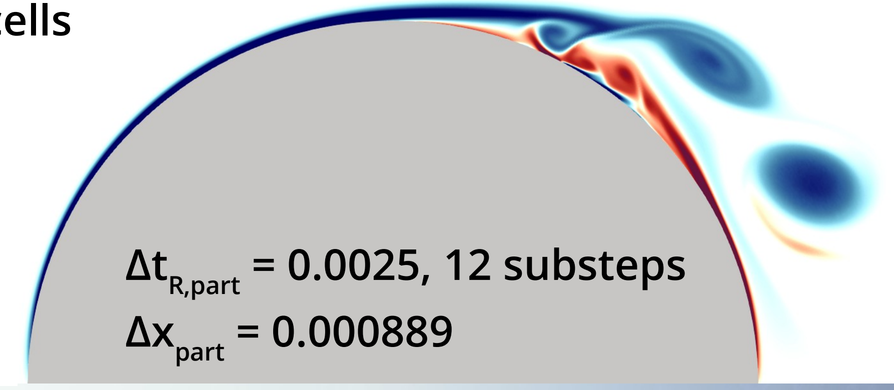




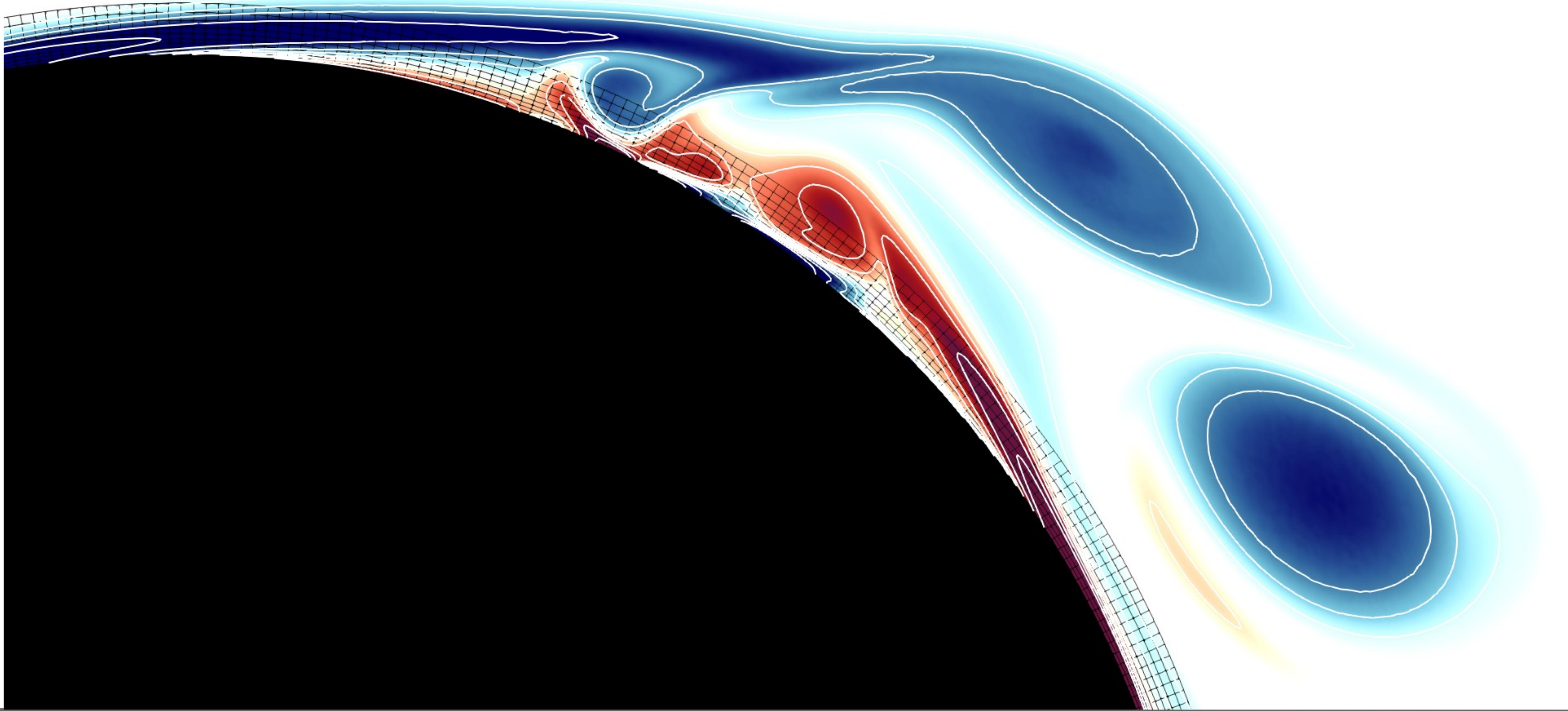

All use mesh to 1.05R
780x7 3rd order cells



# Re=9500 Cylinder





# Conclusions



- First high-order hybrid Lagrangian-Eulerian method
- Retains accuracy in high-gradient near-wall regions
- Requires much smaller Eulerian regions
- Open source package available at:
  https://github.com/Applied-Scientific-Research/Omega2D

# Next Steps

- 3-D Implementation is already in progress
- Investigate limits of Eulerian region thickness and cell size/shape
- Speed up grid-to-particle transfer
- Support multiple moving bodies

# Acknowledgements

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